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BOUNDARY-LAYER RECEPTIVITY TO ACOUSTIC DISTURBANCES

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It is known that, in the case of small external disturbances, boundary-layer transition from laminar to turbulent flow is caused by the growth of unstable Tollmien—Schlichting (TS) waves [1, 2]. The location of transition region and the nature of transition process essentially depend on boundary-layer receptivity to external disturbances, i.e., on the excitation of TS waves by background noise. Scattering of acoustic waves in spatial flow nonuniformities due to surface roughness or nonuniform boundary conditions (nonuniform wall heating, local mass transfer through porous surface, etc.) is a typical mechanism of unstable wave generation.

Excitation of TS waves by streamwise acoustic wave on a small isolated roughness on a flat plate was experimentally investigated in [3]. Asymptotic analysis of this problem was carried out in [4] for the case when the roughness was located in the neighborhood of the lower branch of the neutral curve. Generation of TS waves by sound on sinusoidal and distributed waviness of the flat-plate surface was considered in [5] at small freestream Mach numbers.

Theoretical investigation of the excitation of TS waves by acoustic disturbance on local three-dimensional roughness in a compressible boundary layer is presented in this present paper. Analysis is carried out by reducing the problem to the solution of a system of eigenfunctions of the linearized Navier—Stokes equations [6, 7]. The generation of unstable waves is the result of weak nonlinear interaction of sound with the flow nonuniformity. Computations on the excitation of TS waves on individual roughness element in the flat-plate boundary layer agree well with experimental data [3].

If isolated roughnesses are small, or if they are far from the point of instability so that the final amplitudes of the generated TS waves are small, then it is possible that the distributed generation of unstable waves may become dominant. In this case the excitation is caused by acoustic scattering on weak nonuniformity due to nonparallel flow conditions in the boundary layer [8-10]. A comparison of the effectiveness of TS-wave generation by sound on isolated roughness and on distributed roughness is given in this paper.

1. Consider two-dimensional compressible flow. A roughness element in the form of a small hump, which is a stationary disturbance source in the boundary layer, is located at a disturbance L from the leading edge of the flat plate. External acoustic wave with

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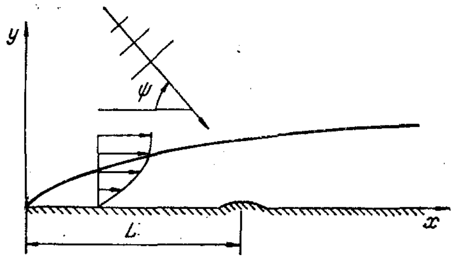

Fig. 1

specified amplitude and frequency is introduced at an angle ψ to the flat plate and is scattered at the roughness exciting TS waves of the same frequency. It is required to determine the characteristics of the generated TS waves.

Consider the coordinate system shown in Fig. 1. Let the reference length scale along the x axis by L, along the y axis, $\delta = (\nu_\infty L/U_\infty)^{1/2}$, and the reference time be δ/U_∞ (ν_∞ and U_∞ are the kinematic viscosity and the velocity of the free stream). Analysis is limited to two-dimensional disturbances. The flow field is described by the vector-function

$$\Psi(x, y, t) = \left(u, \frac{\partial u}{\partial y}, v, p, \theta, \frac{\partial \theta}{\partial y}, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial \theta}{\partial x}\right),$$

where u and v are the x and y components of velocity normalized with respect to U_∞; p is the normalized pressure with reference to $\rho_\infty U_\infty^2$ (ρ is the density); θ is the normalized temperature with reference to the free stream temperature T_∞.

The disturbance flow field is expressed in the form

$$\Psi(x, y, t) = \mathbf{Q}(x, y) + h\mathbf{q}(x, y) + \mathrm{Re}\,[\mathbf{A}(x, y)e^{-i\omega t}]. \tag{1.1}$$

Here Q(x, y) describes the mean flow in the boundary layer on a smooth surface and varies along x by the length scale L; A(x, y) is the amplitude of unsteady disturbances, including the external acoustic wave; q(x, y) corresponds to stationary disturbance due to the hump, localized by the length scale εL; $\varepsilon = R^{-1} = (\nu_\infty/U_\infty L)^{1/2} \ll 1$.

The roughness shape is given by the equation

$$y_w(s) = hf(s), \quad s = \varepsilon^{-1}(x - x_*), \quad f(s) = O(1)$$

($x_* = 1$ is the coordinate of the center of the hump). It is assumed that the roughness height h is much less than the thickness of the viscous sublayer. Then the disturbance due to the roughness is described by linearized Navier–Stokes equations [4, 11].

Substituting Eq. (1.1) in Navier–Stokes equations and retaining terms of the order of ε, h for the unsteady disturbances and terms of the order h for steady disturbance, we get

$$\frac{\partial}{\partial y}\left(L_0 \frac{\partial \mathbf{A}}{\partial y}\right) + L_1 \frac{\partial \mathbf{A}}{\partial y} = H_1\mathbf{A} + \varepsilon H_2 \frac{\partial \mathbf{A}}{\partial x} + \varepsilon H_3\mathbf{A} + hH_4(\mathbf{q})\,\mathbf{A}; \tag{1.2}$$

$$\frac{\partial}{\partial y}\left(L_0 \frac{\partial \mathbf{q}}{\partial y}\right) + L_1 \frac{\partial \mathbf{q}}{\partial y} = H_1\mathbf{q} + \varepsilon H_2 \frac{\partial \mathbf{q}}{\partial x}. \tag{1.3}$$

Matrices L_0, L_1, H_1, and H_2 are 9 × 9 and depend on the mean flow, frequency, and on x as a parameter. Their explicit form is given in [7]. In (1.3) matrix elements are computed for $\omega = 0$. The operator H_3 contains terms resulting from nonparallelness of the mean flow. The operator H_4 describes nonlinear interaction of the steady q and unsteady A disturbances.

The boundary conditions for insulated, highly conductive surface are

$$\Psi_1(x, y_w, t) = \Psi_3(x, y_w, t) = 0, \; \Psi_5(x, y_w, t) = T_{wa}, \tag{1.4}$$

where T_{wa} is the adiabatic wall temperature. Expanding (1.4) in a series in the neighborhood of y = 0, we get of order O(h) + O(ε)

$$A_1(x, 0) = A_3(x, 0) = A_5(x, 0) = 0; \tag{1.5}$$

$$q_1(x, 0) = -U'_w f(s), \quad q_3(x, 0) = q_5(x, 0) = 0, \quad U'_w = Q_2(x, 0). \tag{1.6}$$

As $y \to \infty$, the unsteady disturbances are assumed to be bounded and the stationary disturbances are assumed damped:

$$|\mathbf{A}(x, y)| < \infty, \; y \to \infty; \tag{1.7}$$

$$|\mathbf{q}(x, y)| \to 0, \; y \to \infty. \tag{1.8}$$

The initial condition is specified at the section x_0 located sufficiently far upstream of the local roughness

$$\mathbf{A}(x_0, y) = \mathbf{A}_0(y) \tag{1.9}$$

[$\mathbf{A}_0(y)$ is the acoustic-wave amplitude].

It is assumed that the stationary disturbance $\mathbf{q}(x, y)$ is localized near the roughness and is damped upstream and downstream. Thus, in order to describe the nonstationary field $\mathbf{q}$ from Eqs. (1.3), (1.6), and (1.8), and solve the combined problem (1.2), (1.5), (1.7), and (1.9). Such a formulation corresponds to the first approximation for the weakly nonlinear interaction of disturbances.

2. The amplitude of nonstationary disturbance $\mathbf{A}$ is expressed in the biorthogonal system of eigenfunctions $\{\mathbf{A}_\alpha, \mathbf{B}_\alpha\}$ of the locally homogeneous problem [7] which is obtained from (1.2) if the third and second terms on the right-hand side are neglected:

$$\mathbf{A}(x, y) = \sum_\alpha{}' c_\alpha(x)\, \mathbf{A}_\alpha(x, y) \exp \varphi_\alpha(x), \quad \varphi_\alpha = i \int_{x_0}^{x} \varepsilon^{-1} \alpha \, dx$$

($\sum_\alpha{}'$ denotes summation according to the discrete spectrum and integration is for the continuous spectrum). Here and subsequently, eigenvalues α are nondimensionalized with respect to the reference length δ. Spectral analysis and properties of biorthogonal systems are given in [7]. The following orthogonality conditions are fulfilled:

$$\langle H_2 \mathbf{A}_\alpha, \mathbf{B}_\beta \rangle = \Delta_{\alpha\beta}, \quad \langle H_2 \mathbf{A}, \mathbf{B} \rangle = \int_0^\infty \sum_{i,j=1}^{9} H_2^{ij} A_j \bar{B}_i \, dy, \tag{2.1}$$

where $\mathbf{B}_\alpha$ is the conjugate of $\mathbf{A}_\alpha$; $\Delta_{\alpha\beta}$ is the Kronecker delta when at least one of the eigenfunctions belongs to the discrete spectrum; $\Delta_{\alpha\beta} = \delta(\alpha - \beta)$ is a delta function if α and β belong to the continuous spectrum; the bar denotes complex conjugate.

Consider bimodal condition describing the interaction of acoustic wave with wave number α_A and TS wave with eigenvalue α_{TS}:

$$\mathbf{A} = c_A(x)\, \mathbf{A}_A(x, y)\, e^{\varphi_A(x)} + c_{TS}(x)\, \mathbf{A}_{TS}(x, y)\, e^{\varphi_{TS}(x)}. \tag{2.2}$$

Substituting (2.2) in (1.2) and using the orthogonality condition (2.1),

$$\begin{gathered} \frac{dc_{TS}}{dx} = c_{TS} W_{TS,TS} + \varepsilon^{-1} h c_{TS} W_{h,TS,TS} + c_A W_{A,TS} e^{\varphi_A - \varphi_{TS}} + \varepsilon^{-1} h c_A W_{h,A,TS} e^{\varphi_A - \varphi_{TS}}, \\ \frac{dc_A}{dx} = c_A W_{A,A} + \varepsilon^{-1} h c_A W_{h,A,A} + c_{TS} W_{TS,A} e^{\varphi_{TS} - \varphi_A} + \varepsilon^{-1} h c_{TS} W_{h,TS,A} e^{\varphi_{TS} - \varphi_A}, \\ c_{TS}(x_0) = 0, \; c_A(x_0) = c_{A0} = \langle H_2 \mathbf{A}_0, \mathbf{B}_A \rangle. \end{gathered} \tag{2.3}$$

Here $W_{\alpha\beta} = -\langle H_2 \partial_x \mathbf{A}_\alpha, \mathbf{B}_\beta \rangle - \langle H_3 \mathbf{A}_\alpha, \mathbf{B}_\beta \rangle$ are matrix elements that describe the internodal interaction due to nonparallel effects of the mean flow [8, 10]; $W_{h,\alpha,\beta} = -\langle H_4(\mathbf{q}) \mathbf{A}_\alpha, \mathbf{B}_\beta \rangle$ are matrix elements that describe the internodal interaction as a result of local nonuniformity in flow.

After a formal integration of the equations for $c_{TS}(x)$ from (2.3), we get

$$c_{TS}(x) = \left[\int_{x_0}^{x} \left(c_A W_{A,TS} e^{\varphi_A - \varphi_{TS}} + h \varepsilon^{-1} c_A W_{h,A,TS} e^{\varphi_A - \varphi_{TS}} \right) \exp\left(-\int_{x_0}^{x} E\, dx \right) dx \right] \exp\left(\int_{x_0}^{x} E\, dx \right), \tag{2.4}$$

where $E = W_{TS,TS} + \varepsilon^{-1}hW_{h,TS,TS}$ is the distortion of the eigenvalue of TS wave due to nonparallelness of the mean flow and local roughness.

The roughness shape and stationary disturbance are expressed in the form of Fourier integrals:

$$f(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \rho(\alpha_v)\, e^{i\alpha_v s}\, d\alpha_v,$$

$$\mathbf{q}(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \rho(\alpha_v)\, \mathbf{A}_v(x, y)\, e^{i\alpha_v s}\, d\alpha_v, \quad s = \varepsilon^{-1}(x - x_*).$$

The Fourier component A_v is the solution of the problem on the locally uniform flow in x:

$$\frac{d}{dy}\left(L_0 \frac{d\mathbf{A}_v}{dy}\right) + L_1 \frac{d\mathbf{A}_v}{dy} = H_1 \mathbf{A}_v + i\alpha_v H_2 \mathbf{A}_v, \tag{2.5}$$
$$A_{v1} = -U_w', \quad A_{v3} = A_{v5} = 0, \quad y = 0, \quad |\mathbf{A}_v| \to 0, \quad y \to \infty.$$

Here the matrix elements L_0, L_1, H_1, H_2 are computed at $\omega = 0$.

Since the operator H_4 is linearly dependent on q, the following relation is valid:

$$W_{h,A,\mathrm{TS}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \rho(\alpha_v)\, V_{v,A,\mathrm{TS}}\, e^{i\alpha_v s}\, d\alpha_v, \tag{2.6}$$

where $V_{v,A,TS}(\alpha_v, A_v)$ is the matrix element that describes the nonlinear interaction between stationary disturbance with wave number α_v and the acoustic wave. Neglecting the first term in the integral expression (2.4) which is responsible for the excitation of TS waves due to nonparallelness of the mean flow, and substituting (2.6) in (2.4), we get

$$c_{\mathrm{TS}}(x) = \frac{h}{2\pi}\int_{-\infty}^{\infty} \rho(\alpha_v)\, I(\alpha_v)\, d\alpha_v \exp\left[\int_{x_0}^{x} E\, dx\right]; \tag{2.7}$$

$$I(\alpha_v) = \varepsilon^{-1}\int_{x_0}^{x} c_A V_{v,A,\mathrm{TS}}\, e^{\varphi_A - \varphi_{\mathrm{TS}} + \varphi_v} \exp\left[-\int_{x_0}^{x} E\, dx\right] dx, \tag{2.8}$$
$$\varphi_v = i\varepsilon^{-1}\int_{x_*}^{x} \alpha_v\, dx.$$

Let the coordinate of the center of the hump x_* coincide with the point of instability x_{pi} for TS waves (analysis for roughness displaced from the point x_{pi} is given below). Then resonance of excitation is realized at the point x_* for $\alpha_v \equiv \alpha_{v*} = \alpha_{TS} - \alpha_A$. The large parameter ε^{-1} as the index of the exponent in the integral expression (2.8) makes it possible to determine the asymptote of the integral $I(\alpha_v)$ using the method of steepest descent. Downstream of the hump for $x - x_* \gg \varepsilon^{1/2}x_*$, $\varepsilon \to 0$

$$I(\alpha_{v*}) = c_A(x_*)\, g \exp\left[i\int_{x_0}^{x_*} \varepsilon^{-1}(\alpha_A - \alpha_{\mathrm{TS}})\, dx - \int_{x_0}^{x_*} E\, dx\right], \tag{2.9}$$
$$g = \left(\frac{2\pi}{\varepsilon\left|\frac{d\alpha_{\mathrm{TS}}}{dx}\right|_*}\right)^{1/2} V_{v,A,\mathrm{TS}}(\alpha_{v*}, x_*)\, e^{i\varphi} + O(\varepsilon^{1/2}).$$

Here the asterisk denotes quantities computed at x_*, φ is a real constant determined by the choice of the square root.

In the event of a deviation from resonance $\Delta\alpha = \alpha_v - \alpha_{v*}$ the saddle point z_*, determined from the equation $\alpha_A - \alpha_{TS} + \alpha_v = 0$, will be complex:

$$z_* = x_* + \frac{\Delta\alpha}{\left(\frac{d\alpha_{TS}}{dx}\right)_*} + O(\Delta\alpha^2).$$

Expanding the subintegral functions $c_A(x)$, $V_{v,A,TS}(x)$ in a small neighborhood of the complex plane z, enclosing $\sqrt{\varepsilon}$ region of the saddle point, assuming that these are analytic functions. Integrate (2.8) along the contour passing through the saddle point along the line of steepest descent, and expand the result of integration in series in the neighborhood of the center of the hump x_*. Then, as $\varepsilon \to 0$, $x - x_* \gg \varepsilon^{1/2} x_*$,

$$I(\alpha_v, x) = I(\alpha_{v*}) \exp\left[\frac{i}{2\varepsilon} \frac{(\alpha_v - \alpha_{v*})^2}{\left(\frac{d\alpha_{TS}}{dx}\right)_*}\right]. \tag{2.10}$$

Substituting (2.10) in (2.7) and integrating for all α_v, we get an asymptotic expression for the amplitude of the generated TS wave

$$\Phi_{TS}(x, y) = h a_A(x_*) \rho(\alpha_{v*}) V_{v,A,TS}(\alpha_{v*}, x_*) \mathbf{A}_{TS}(x, y) e^{F_{TS}}, \tag{2.11}$$

$$a_A(x_*) = c_{A0} \exp\left[i \int_{x_0}^{x_*} \varepsilon^{-1} \alpha_A dx\right], \quad F_{TS} = \int_{x_*}^{x} (i\varepsilon^{-1}\alpha_{TS} + E)\, dx$$

[$a_A(x_*)$ is the amplitude of sound wave at resonance]. If the hump is displaced from the point of instability, α_{v*} will be complex. In completing the integral in Eq. (2.7) it is necessary to continue the subintegral function into the complex region α_v enclosing the $\sqrt{\varepsilon}$ region of the saddle point α_{v*}, and to use the method of steepest descent. The first approximation in ε again reduces to Eq. (2.11).

Thus, the excitation of TS waves is localized in the segment $|x - x_*| \sim \varepsilon^{1/2} x_*$ and takes place in a narrow range of wave numbers $|\alpha_v - \alpha_{v*}| \sim \varepsilon^{1/2} \alpha_{v*}$. As the wave number α_v moves away from the resonant value, the amplitude of TS waves is exponentially decreased.

3. It follows from (2.11) that the amplitude of maximum fluctuations along y of the streamwise component of mass flow in the excited TS wave

$$q_m(x) = h\rho(\alpha_{v*}) |a_A(x_*)| P_{A,TS}(\alpha_{v*}, x_*) \frac{q_{TS}(x)}{q_{TS}(x_*)} \exp(\operatorname{Im} F_{TS}); \tag{3.1}$$

$$P_{A,TS} = \left| \frac{V_{v,A,TS}(\alpha_{v*}, x_*)}{\langle H_2 \mathbf{A}_{TS}, \mathbf{B}_{TS} \rangle_*} \right| q_{TS}(x_*), \tag{3.2}$$

where q_{TS} is the modulus of the x component of the mass flow computed from the vector A_{TS} at the point of maximum along y; $P_{A,TS}$ is the coupling coefficient of the acoustic wave and TS wave which characterizes the effectiveness of the excitation mechanism. The introduction of the factor $\langle H_2 A_{TS}, B_{TS} \rangle^{-1}$ ensures invariance of (3.2) relative to the choice of normalization of eigenfunctions A_{TS}, B_{TS}. In the normalization of the acoustic mode A_A is fixed, the coupling coefficient does not depend on the form of the hump and the strength of the acoustic wave. Its value is equal to the amplitude of TS wave excited by the acoustic wave of unit amplitude [$a_A(x_*) = 1$] on a hump of height h = 1 and with resonant harmonic $\rho(\alpha_{v*}) = 1$.

The coupling coefficients $P_{A,TS}$ were computed for boundary layer on a flat plate with the adiabatic index equal to 1.41, Prandtl number 0.72, and stagnation temperature 310°K. The

TABLE 1

M	R	$\alpha_{TS} \cdot 10^2$	$\alpha_{v*} \cdot 10^2$	$P_{A,TS} \cdot 10^2$	R	$\alpha_{TS} \cdot 10^2$	$\alpha_{v*} \cdot 10^2$	$P_{A,TS} \cdot 10^2$
	$F=20 \cdot 10^{-6}$				$F=60 \cdot 10^{-6}$			
0,2	1020	7,27	6,95	6,50	550	10,04	9,52	6,37
0,4	1010	7,02	6,47	4,90	540	9,64	8,76	4,90
0,6	960	6,44	5,74	3,42	520	8,97	7,84	3,54
0,8	900	5,73	4,96	2,14	490	8,06	6,88	2,33

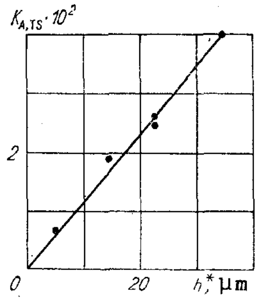

Fig. 2

free stream Mach number was varied in the range 0.2-0.8. Sutherland's formula was used for computing viscosity. Eigenfunctions and matrix elements were computed using Dunn—Lin approximation [12]. Acoustic mode was specified by the conditions: angle of incidence ψ = 20°, amplitude of streamwise velocity fluctuations in the incident wave $u_A = 1$ at $x = x_*$. Computed results are given in Table 1. Reynolds number R, eigenvalues of TS waves α_{TS} and resonant wave numbers α_{v*} are also given there. It follows from Table 1 that at subsonic speeds the effectiveness of excitation weakly depends on the frequency parameter $F = \omega\nu_\infty/U_\infty^2$.

A comparison with experimental results [3] has been carried out to verify theory. The experiment was conducted on a flat plate boundary layer with a free-stream velocity of U_∞ = 23.4 M/sec. The hump was located at a distance L = 0.565 m from the leading edge, had a triangular shape with a streamwise dimension ℓ = 12 mm (approximately a quarter of TS wavelength). Its height was varied in the range h^* = 5-35 μm. Plane, streamwise acoustic wave was beamed at the boundary layer with a frequency of 138 Hz and propagating upstream, ψ = 180°. The test conditions for the experiment were: M = 0.066, R = 901, $F = 25.4\cdot10^{-6}$, $\alpha_A = -1.62\cdot10^{-3}$, $\alpha_{TS} = 7.93\cdot10^{-2}$. The amplification factor $K_{A,TS}$ equal to the ratio of the amplitude of the fluctuation streamwise velocity in TS wave measured at the point of maximum along y to the corresponding amplitude of acoustic wave at the same point, was computed. A comparison of theoretical computations (continuous line) and experiment results (points) is shown in Fig. 2. The scatter does not exceed 11% and lies within experimental measurement accuracy.

It is worth noting that the equation for the amplitude of excited TS waves (2.10) depends only on local flow characteristics along x and is valid for attached boundary layers on bodies with characteristic length scale ~L. The given equation makes it possible to compute the excitation of TS wave by sound on nonuniformities caused by local heating of the wall or local suction through a porous surface. In this case, it is necessary to replace boundary conditions in (2.5) to compute resonant Fourier-harmonics of stationary disturbance.

If the local nonuniformity on the surface of the body is vanishingly small, the dominant element in Eq. (2.4) is $W_{A,TS}$ which describes the generation of TS waves by sound on the distributed nonuniformity due to nonparallel flow effects in the boundary layer. The given type of excitation is discussed in [8-10]. Computations were carried out for flat plate boundary layer at M = 0.6 to compare the effectiveness of the generation of TS wave on local nonuniformity and on nonparallelness of the mean flow. The external acoustic wave had the parameters: $F = 20\cdot10^{-6}$, ψ = 20°. The computation of distributed generation was carried out using the algorithm described in [9]. The local excitation was computed for a nonuniformity located at the point of loss of stability (R = 960) and having a form with resonant Fourier-harmonic $\rho(\alpha_{v*}) = 1$. Computations showed that excitation on distributed roughness due to a slow growth of boundary layer along the flow is equivalent to the excitation on local roughness having characteristic dimension $h^* \simeq 10^{-3}\delta$. Thus, for boundary layer with δ = 0.6 mm, which corresponds to experimental condition [3], the equivalent roughness at the point of loss of stability $h^* \simeq 0.6$ μm. As expended, the distributed generation of TS wave is significantly weaker than the generation on isolated flow nonuniformity. This is explained by the fact that the scale of flow nonuniformity on a smooth surface L appreciably exceeds the scale of intermodal exchange $\ell \sim \varepsilon L/|\alpha_A - \alpha_{TS}|$. The subintegral function in Eq. (2.4) rapidly oscillates, and the integral has exponentially small values.

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DYNAMICS OF LAMINAR VORTEX RINGS IN A STRATIFIED LIQUID

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UDC 532.527

The study of isolated vortices and interacting vortical structures on different scales (the principal structural elements in developed turbulence) is a traditional problem of fluid dynamics. In recent years there has been substantial progress in explaining the nature of the stability of vortices resulting from the stabilizing effect of centrifugal forces which suppress transport in the radial direction [1]. It has been established experimentally that there is a laminar core inside turbulent vortex rings [2]. A survey of theoretical and experimental studies of the motion of vortices in a uniform fluid was made in [3]. The dynamics of an isolated vortex is determined to a significant extent by the involvement of the surrounding fluid in the circulating motion and the loss of vorticity in the wake.

The question of the stability and evolution of a vortex in a stratified fluid is more complex. In this case, centrifugal forces are jointed by buoyancy, which suppresses motion in the vertical direction. Most experimental studies have investigated the vertical motion of vortex rings in a nonuniform fluid [4], modeling the motion of thermals in a stratified atmosphere [5, 6], vortex cores behind an airplane wing [7, 8], and structural elements of free turbulent flows [9]. The authors of [10] visualized a laminar vortex ring moving along the interface of mixing fluids. The interaction of an obliquely-moving vortex ring with a

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